MAS 08 Optimum Joint Event and Parameter Estimation in SN Based on Random Set Theory

MAS 08.1 People
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MAS 08.2 Overview
Recently, there has been a continuing increase in popularity and interest to engineer small low cost devices that are capable of sensing, data processing and communication in wireless sensors. By virtue of its cost, a wireless sensor network (WSN) allows the possibility of monitoring complex scenarios with a finer spatial resolution (i.e., more devices per area). As demand for power efficiency and complexity continues to grow, research and development continue to search for better ways to manage, analyze and combine information that changes rapidly and may be prone to errors. Detecting, localizing, and tracking targets are different aspects of monitoring that have wide interests from bio-complexity study of animals to military surveillance. In reality, targets may enter and exit the monitored area and can sometimes fail to exhibit (either purposely or accidentally) their signals due to lack of power, bad propagation conditions, or masking by noises and interferences. The sensors can also fail from lack of power, hardware failure, software failure, or any combinations of these. In the past, there are difficulties to model these problems using standard probability theory. We propose the use of a new random finite set theory (RFST) approach. RFST is a theory developed on random sets, which generalizes probability theory to set domain. The flexible structure of a set in RFST makes it easier to model variation in both the number of elements and the value of the elements jointly. By working in the set domain, a no-target case can easily be modeled by an empty set, which often can have non-zero probability. It can also model sensor failures, lost connections, noises, and clutters. In addition, RFST allows us to treat both multi-target and multi-sensor jointly as a single random set that has a set density in a similar way as a random variable with its probability density. Optimization done jointly over all the possible randomness yields better system performance than optimizations performed sequentially on different components of the randomness. For example, it allows one to define the expectation of a function, a maximum likelihood estimate (MLE), the Bayesian recursive prediction-filter form, and even the Cramér-Rao Lower Bound (CRB), and many other statistical concepts proven to be useful in estimation and tracking theories based on standard probability theory.

MAS 08.3 Approach
To simplify our discussion, we will assume a uniform grid sensor array deployment over a region of interest (ROI). Each sensor is a self-contained battery operated small computer that has a wireless connectivity for communication to a fusion center. These type of sensors have begin to emerge in the market and will become more readily available over time. Being battery operated, power management is a major challenge. Since wireless communication takes much power, the overall system will benefit greatly if we can minimize communication as much as possible.

Our following discussion is organized into the following sections. We will begin the next two sections defining the observation model and the motion model in terms of random set. Then we will dedicate to the derivation of the set densities from the model. Finally, we will consider some simulation results of a scenario for target tracking and the importance of detecting sensor failure in the system.
1. Observation Model

Let the set \( \Sigma \) denotes the collective observations received at the fusion center. Given \( N \) sensors, we can express \( \Sigma \) as \( \bigcup_{i=1}^{N} (\Sigma_i \times i) \), where \( \Sigma_i \) is the observation from sensor \( i \), and \( i \) is the index. Since the internal sensor noise is independent between sensors, the probability set density for \( \Sigma \) becomes \( f_{\Sigma} = \prod_{i=1}^{N} f_{\Sigma_i} \). To describe the event for each sensor \( i \), we write

\[
\Sigma_i = \Sigma_i^c \cap F_i, \quad (1),
\]

\[
\Sigma_i = \{Z_i\} \cap D_i, \quad (2),
\]

where \( \Sigma_i^c \) is the sensor's observation, \( F_i \) is the sensor failure state, and \( D_i \) is the sensor detection state. \( Z_i \in R \) is the detected signal power with form \( Z_i = h_i(\Theta) + W \), where \( W \) is a random noise with density \( f_W(w) \), and \( \Theta \) is a random set describing the target geo-kinematic properties. Supposed that \( M \) is the measurement space, then we can choose a discrete random subset \( D_i \) of \( M \) such that \( D_i = \emptyset \) with probability \( 1 - p_d \) and \( D_i = M \) with probability \( p_d \). The sensor failure state can also be written in the similar manner to describe failing state \( (F_i = \emptyset) \) or normal state \( (F_i = M) \).

To conserve power, we define a threshold \( \tau \), such that

\[
\Sigma_i = \{\emptyset, \text{if } Z_i < \tau; \{Z_i\}, \text{if } Z_i \geq \tau\}, \quad (3).
\]

This will limit the sensor's communication to the fusion center only when it is reasonably significant. Then from (2) and (3), \( D_i \) is described completely by the thresholding (i.e. \( p_d = P(Z_i \geq \tau) \)). The function \( h(.) \) describes the relationship between the target position and the detected signal amplitude. In our case, we assume that the source emits a signal with power \( P_0 \) when measured at a reference distance \( d_0 \) in a free space, which is given by

\[
h_i(\emptyset) = 0, \quad (4)
\]

\[
h_i(\Theta) = \sqrt{P_0 d_0^2 / d^2(\Theta_i, \Theta)}, \quad (5),
\]

where we denote \( h_i(\Theta)h_i(\Theta_i) \), \( \Theta = [x, y, u_x, u_y]^T \) is the target position and velocity, \( \Theta_i = [x_i, y_i]^T \) is sensor \( i \) position, and \( d(\Theta_i, \Theta) = \sqrt{(x-x_i)^2 + (y-y_i)^2} \) is the distance between the sensor \( i \) and the target.

Throughout this article \( ^T \) will denote the transpose operation, and we will use \( \Theta \) as the shorthand notation for the set \( \{\Theta \} \) to reduce notation clutter.

For sequential indexing, we will denote the time by augmenting the subscript with a time index \( t \), so \( \Sigma_{i,t} \) will denote the observation of sensor \( i \) at time \( t \). We will further assume that the signal is received by the farthest sensor before the next epoch.
2. Motion Model

When a target exists, we assume it follows a constant velocity with a small random perturbation and a density of \( f_y(v) \). For each epoch, we want to update the \( \Theta \) and all the \( F_i \). The update equation can then be written as

\[
\Theta_{t+1} = S^\Theta(\Theta_t) \cup B^\Theta(\Theta_t),
\]

(6),

\[
S^\Theta(\emptyset) = \emptyset,
\]

(7),

\[ S^\Theta(\Theta_t) = \left\{ \emptyset, \text{ with probability } 1 - p_s; \ s(\Theta_t), \text{ with probability } p_s \right\}. \]

(8),

\[
B^\Theta(\emptyset) = \left\{ \emptyset, \text{ with probability } 1 - p_b; \ b, \text{ with probability } p_b \right\},
\]

(9),

\[
B^\Theta(\Theta_t) = \emptyset,
\]

(10),

\[ s(\Theta_t) = \left[ x + u_x + v_x; y + u_y + v_y; u_x + v_x; u_y + v_y \right]. \]

(11),

where \( B^\Theta \) and \( S^\Theta \) describe the birth and the surviving set function respectively. If a target survives, \( s(\Theta_t) \) describes its state transition behavior. If a birth occurs, \( b \) describes the geo-kinematic probability of the target when it appears with the density \( f_b(b) \). \( x \) and \( y \) denotes the position of the target, and \( u_x \) and \( u_y \) denotes its component velocities. \( v_x \) and \( v_y \) are the random perturbation, which is described by \( f_y(v) \).

For each \( F_{i,t} \), the update equation is given by

\[
F_{i,t+1} = S^F(F_{i,t}, \Theta) \cup B^F(\Sigma_{i,t}),
\]

(12),

\[
B^F(\Sigma_{i,t}) = \left\{ \emptyset, \text{ if } \Sigma_{i,t} = \emptyset; M, \text{ if } \Sigma_{i,t} \neq \emptyset \right\},
\]

(13),

\[
S^F(-) = \left\{ q_i(\Theta), \text{ if } F_{i,t} = M, \Theta = \emptyset; F_{i,t}, \text{ otherwise, } \right\},
\]

(14),

where \( S^F \) models the sensor survival, and \( B^F \) is the set indicator modeling the sensor birth (or recovery from failure). \( q_i(\Theta) \) is a random set that models the decision to tag sensor failure when an estimate exists and sensor \( i \) provides no detection. Intuitively, if a target passes very close without any report, the sensor has probably failed, and a sensor far away from the target should not be tagged as failing. To describe this, we set \( q_i(\Theta) = \emptyset \) if \( h_i(\Theta) < \tau' \), where \( \tau' = \tau + \alpha \sigma \), and \( \alpha \geq 0 \). Under normal conditions, each sensor’s observation is perturbed by an internal noise with variance \( \sigma^2 \). We want to set \( \alpha \) so that \( S^F \) is insensitive to most internal noise perturbation. If we set \( \alpha = 0 \), all sensors that failed to report will be incorrectly tagged as failing. \( \alpha \) around 2 - 3 are reasonable values to ignore most sensor noise perturbation. In this way, only an unreasonably large deviation will activate the sensor failure tagging.

3. Observation Densities

Before we can use the model defined in Sec. 1, we will need to derive the observation set densities. Recall that the belief function is defined as \( \hat{\beta}_Z(C)P(Z \subseteq C) \) and the measurement space \( M \) is such that \( \hat{\beta}_Z(M) = 1 \). We begin by writing the belief function of (1) as
\[
\beta_{z_{i,t}}(C \mid \Theta_i, \emptyset) = P(\Sigma_{i,t} = \emptyset), \quad (15),
\]
\[
\beta_{z_{i,t}}(C \mid \emptyset, M) = 1 - p_{fa} + p_{fa}P_d, \quad (16),
\]
\[
\beta_{z_{i,t}}(C \mid \Theta_i, M) = 1 - P_{d_i}(\emptyset) + P_{d_i}(\emptyset)P_b, \quad (17),
\]
where the notation \( P_{fa} \) and “false-alarm” probability, is needed to distinguish (16) from (17). By applying set derivatives and evaluating the results at \( C = \emptyset \), the set densities are given by
\[
f_z(\emptyset \mid \Theta_i, \emptyset) = 1, \quad (18),
\]
\[
f_z(z \mid \Theta_i, \emptyset) = 0, \quad (19),
\]
\[
f_z(\emptyset \mid \emptyset, M) = 1 - p_{fa}, \quad (20),
\]
\[
f_z(z \mid \emptyset, M) = p_{fa}f_z(z), \quad (21),
\]
\[
f_z(\emptyset \mid \emptyset, M) = 1 - P_{d_i}(\emptyset), \quad (22),
\]
\[
f_z(z \mid \emptyset, M) = P_{d_i}(\emptyset)f_z(z), \quad (23),
\]
where \( f_z(C \mid A_1, A_2)f_z(C \mid \Theta_i = A_1, F_{i,t} = A_2) \). If a target exists, \( f_z(z) \) is the target's observation probability distribution. Similarly, if a clutter exists, \( f_c(z) \) is the clutter's observation probability distribution.

For the sensor noise distribution, we will assume \( f_w(w) \sim N_w(0, \sigma^2) \). Although this is a simplistic assumption for real applications, it helps to reduce equation complexity. Consequently, \( p_{fa} = Q((\tau - h_i(\emptyset)) / \sigma) \), and \( P_{d_i}(\emptyset) = Q((\tau - h_i(\emptyset)) / \sigma) \). Clearly, more realistic noise pdf can also be used.

4. Motion Densities

Similarly, we also need to derive the set densities from the model described in Sec. 2. The belief function for (6) can be written as
\[
\beta_{\Theta_i}(C \mid \Theta_i = \emptyset) = 1 - p_b + p_bPC, \quad (24),
\]
\[
\beta_{\Theta_i}(C \mid \Theta_i = \emptyset) = 1 - p_s + p_sPD, \quad (25),
\]
where \( PC = P(\Theta_{i,t} \subseteq C \mid C \neq \emptyset, \Theta_i = \emptyset) \) and \( PD = P(\Theta_{i,t} \subseteq C \mid C \neq \emptyset, \Theta_i = \emptyset) \). Following the same procedures, the set densities can be obtained as
\[
f_{\Theta_i}(\emptyset \mid \emptyset) = 1 - p_b, \quad (26),
\]
where \( f_b \) and \( f_s \) are as described in Sec. 1.

For the sensor failure state, we can write the belief functions as

\[
\beta_{F_{i+1}}(C | \cdot) = \begin{cases} 
1 - p_f + p_f p_s & \text{if } F_{i+1} = M, \Theta = \emptyset, B^F = \emptyset; P(F_{i+1} = M), \text{ if } B^F = M; P(F_{i+1} = F_{i+1}), \text{ otherwise. (30)}, 
\end{cases}
\]

where \( P = P(F_{i+1} \subseteq C | C \not= \emptyset, F_{i+1} = M, \Theta = \emptyset, B^F = \emptyset) \). Since \( F_{i+1} \) takes only discrete values, the set probability mass functions are given as

\[
\begin{align*}
\beta_{F_{i+1}}(\emptyset | M, \emptyset, \emptyset) &= p_q(0), \quad (31), \\
\beta_{F_{i+1}}(M | M, \emptyset, \emptyset) &= 1 - p_q(0), \quad (32), \\
\beta_{F_{i+1}}(M | F_{i+1}, \emptyset, M) &= 1, \quad (33), \\
\beta_{F_{i+1}}(\emptyset | F_{i+1}, \emptyset, M) &= 0, \quad (34), \\
\beta_{F_{i+1}}(M | \emptyset, \emptyset, \emptyset) &= 0, \quad (35), \\
\beta_{F_{i+1}}(\emptyset | \emptyset, \emptyset, \emptyset) &= 1, \quad (36), \\
\beta_{F_{i+1}}(M | M, \emptyset, \emptyset) &= 1, \quad (37), \\
\beta_{F_{i+1}}(\emptyset | M, \emptyset, \emptyset) &= 0, \quad (38),
\end{align*}
\]

where \( p_q(\emptyset) = Q(\tau - h_t(\emptyset) / \sigma) \).

**MAS 08.4 Accomplishments**

In a real system, each sensor has a separate processing unit, so the complexity of the whole system is well distributed. To practically simulate the entire sensor suite, we will use particle filtering approach using the sequential importance resampling method.

The region of interest (ROI) is a 200 x 200 square meter with 100 sensors deployed on a uniform grid. For reference, we place the axis \((0, 0)\) at the center of the ROI. The target, when measured at \(d_0 = 1\) meter, emits a signal with power intensity of \(P_0 = 800\) watts. The noise variance for each sensor is \(\sigma^2 = 0.4\), and the \(\tau\) is set to 1.6. Other parameters are given as \(\alpha = 1, p_s = 0.92, p_b = 0.2, f_\nu(\nu) \sim N(0, 1)\).

Let’s consider a scenario where a target repeatedly pass through the same path. When all sensors behave normally, the system is able to track the target properly (e.g. Fig. 1). If some sensors in the proximity of the path fail, the estimate can be very erroneous (e.g. Fig. 2). Normal non-transmitting sensors carry inherent information that the target is far from its position. Hence, when a nearby sensor fails to transmit, the estimate becomes inaccurate. If the sensor failure detection is employed, the system can avoid large errors by tagging sensors that might be failing.
The effect of automatic sensor failure detection can be seen in Fig. 3(a). After the failed sensors are tagged, future tracking efforts can use only the remaining sensors as seen in Fig. 3(b).

Sometimes, the target stops emitting its signal or purposefully hides its presence from the sensors. The analysis on target’s behavior is beyond the scope of this work at this point; however, we still want to detect if this event occurs. In Fig. 4, the system begins to track a target that suddenly disappear. Four time instants later, a target appears at a different starting point. Note that the second trace can be a different target from the first. Notice also that sensors that are mistakenly tagged as failing does not degrade much of the system performance. Once a tagged sensor sends information, the system can immediately use it and simultaneously lift the tag off the sensor. Information is lost only if a working sensor does not send the data due to thresholding, though it is often a negligible amount.

**MAS 08.5 Future Directions**

Simultaneous randomness in the number of targets and sensors along with their geo-kinematic information can be modeled methodically using the RFST method. This modeling system allows a systematic derivation of densities that can be analyzed using Bayesian recursive prediction-filter methodologies. The simulation results have exemplified the importance of sensor failure detection in a real system. When failed sensors incorrectly interpreted as non-reporting working sensor, the tracking estimates show severe degradation. Once the sensor failure detection is operating, the system can quickly avoid large errors by tagging sensor that might be failing. Future targets can then be tracked correctly based on the remaining active sensors. A randomly missing target can also be detected using the same scheme. We also have shown that the system can track the number of targets and simultaneously estimate the geo-kinematic information of the target within the ROI.
In this year, we have demonstrated explicitly by analysis and simulation the advantage of using RFST method to perform joint optimum detection, localization, and tracking when the number of targets and number of sensors can be randomly changing. In the coming year, we will work with real-life data to verify the advantage of the RFST approach over standard ad hoc approaches.

**MAS 08.6 External Research Partnerships**